Trafalgar Junior School
Written Calculation Policy

This Calculation Policy was designed to ensure consistency and progression in teaching and learning throughout Trafalgar Junior School.

It is important to remember that the written methods of addition, subtraction, multiplication and division build upon children's understanding of place value, the relationship between the operations and the ability to perform mental calculations using all four operations. Throughout their primary schooling, children continue work on improving and extending their use of mental calculations and the learning number of facts in order to aid their work on written calculation methods.

The written calculations are shown in this policy as stages and therefore children within the same year group may be at different stages depending on their understanding. Pupils are also encouraged to identify and use the most efficient and best suited method depending on the numbers used within a calculation.

NB

- HTU (hundreds, tens & units) are now known as **HTO** (hundreds, tens and *ones*). This is to avoid confusion when referencing 'units' in other mathematical contexts (such as kg, cm, ml, etc) as well as directly recognising their value as 'ones'.

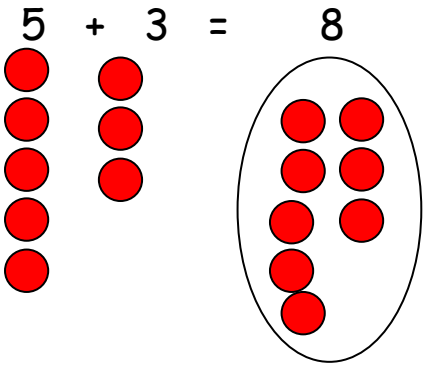
ADDITION

The Early Years Foundation Stage (EYFS) and Key Stage One children are taught mental calculations which are systematically built upon throughout Key Stage Two.

Children are introduced to the processes of calculation through practical, oral and mental activities, such as counting songs, counting by rote and touch counting as well as through experiencing real-life scenarios, such as buying and selling in a shop. This will include language such as: more, add, together, total, altogether, how many.

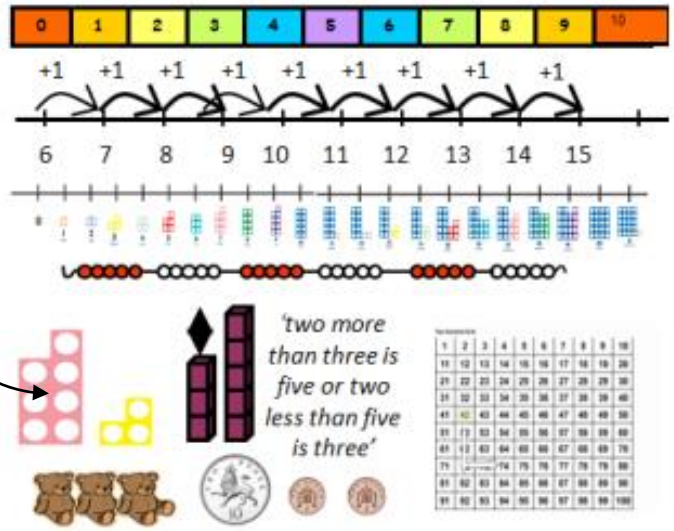
Resources that could be used to support this learning:

Counting all:

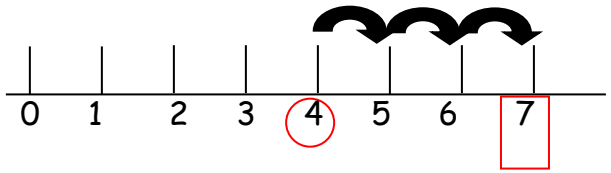
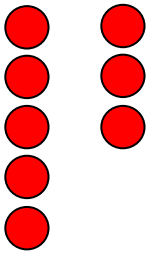
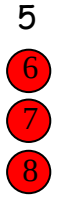
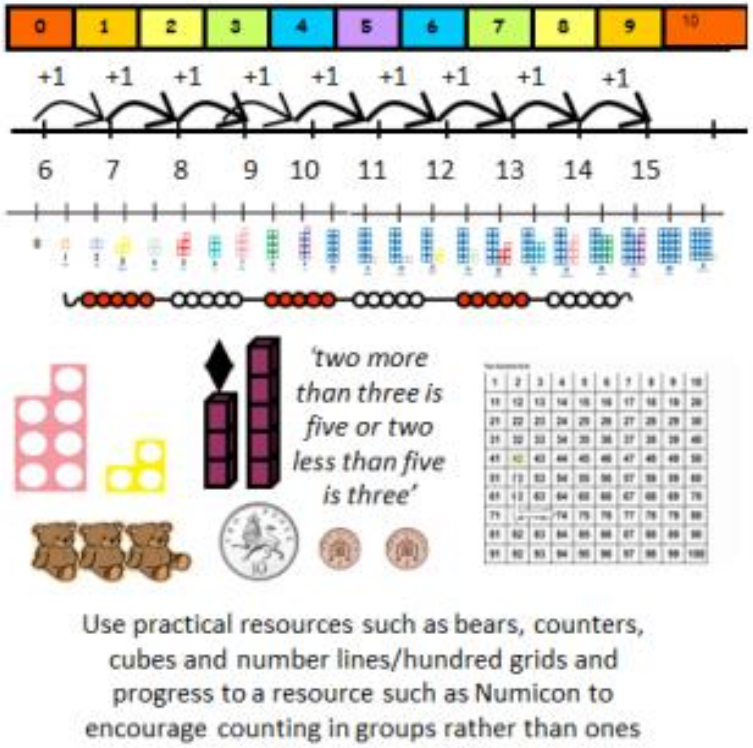


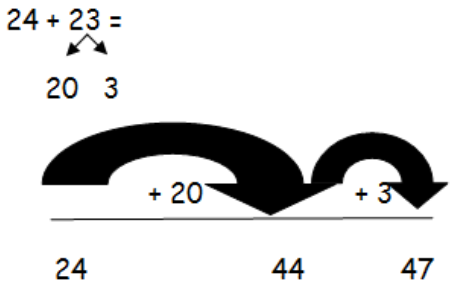
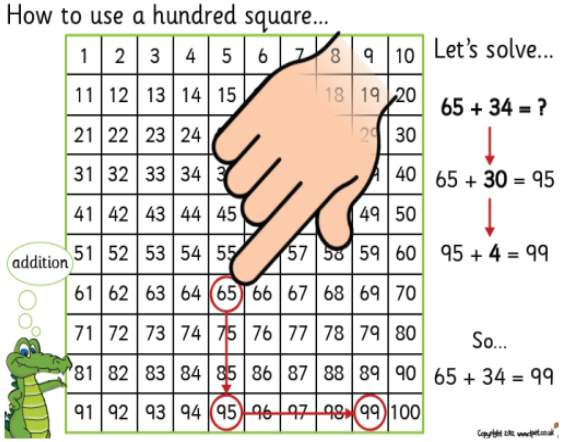
I count out 5 counters and 3 counters.
I put the counters together and count them together.
1, 2, 3, 4, 5, 6, 7, 8.

Numicon



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

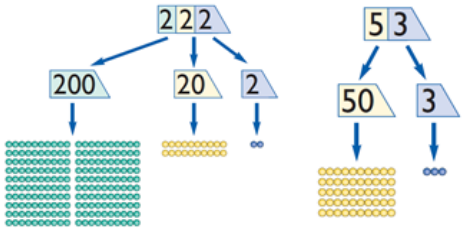
STAGE ONE	
The number line is also supported by the use of a counting stick and bead string. *Other resources which can be used to support children with their understanding of number include Numicon and number squares.* Children are taught to add larger numbers together using the number line as their understanding of number progresses.	<u>Number Line</u> A blank or annotated number line is used for counting on. Children jump on the top of the number line (tapping the numbers on their heads) when adding. For example; 4 + 3 =  Counting on: 5 + 3 = **8**  I count out 3 counters I put the number 5 in my head, Then I count on 6,7,8  *Numicon*  Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones

STAGE TWO	
When children are ready to be adding larger numbers, they will continue to do so on a number line or a bead string. *To support this learning, children will use a hundred square. They will be encouraged to jump down in 10s and forwards in units or ones. Children are encouraged to then record this as a written calculation.* Children will continue to use the blank number line throughout each stage as an efficient method of addition.	Children will be taught to begin with the biggest number and partition the number to be added on. Children jump on top of the number line when adding.  

STAGE THREE

Resources used to support children in the expanded method of column addition (partitioning in a way that leads to a formal method of addition) include place value cards.

Partitioning:
222 + 53 =



2 + 3 = 5
20 + 50 = 70
200 + 0 = 200
200 + 70 + 5 = 275

Children will use the expanded method of column addition for larger numbers (HTO) including adding decimals in the context of money and measure.

<u>Expanded method of column addition</u>

Once children are confident with place value and partitioning, they begin to set out addition as below to support the formal method. Children should start adding from the ones column and work right to left. 1 digit = 1 square.

<u>Note the addition sign on the LEFT hand side.</u>

Children must be taught to leave a row and/or a column of squares for this:

57 + 35 = <u>92</u>

	T				O				
	5	0			7				
+	3	0			5				
	8	0	+	1	2	=	9	2	

368 + 493 = <u>861</u>

	H					T				O				
	3	0	0			6	0			8				
+	4	0	0			9	0			3				
	7	0	0	+	1	5	0	+	1	1	=	8	6	1

STAGE FOUR

Children progress onto learning how to work down the page, rather than across, to support the further learning of the column addition.

124 + 79 = 203

		H	T	O	
		1	2	4	
	+		7	9	
			1	3	
			9	0	
	+	1	0	0	
		2	0	3	
		1			

4+9

20 + 70

100 + 0

STAGE FIVE

When children are secure with place value, they can begin using the compact method of addition with at least one 'carry'.

Column addition

When exchanging, the ten or hundred etc. is placed under the answer box. Children must be taught to leave a row and/or a column of squares for this:

1 digit = 1 square.

Note the addition sign on the LEFT hand side and it leads to good habits if children place the larger number at the top.

247 + 726 = 973

	H	T	O
	7	2	6
+	2	4	7
	9	7	3
		1	

Extend to *bigger numbers*...

34,809 + 19,667 = 54,476

		TTh	Th	H	T	O
		3	4	8	0	9
+		1	9	6	6	7
		5	4	4	7	6
		1	1		1	

...and *decimals*.

35.2 + 16.0 = 51.2

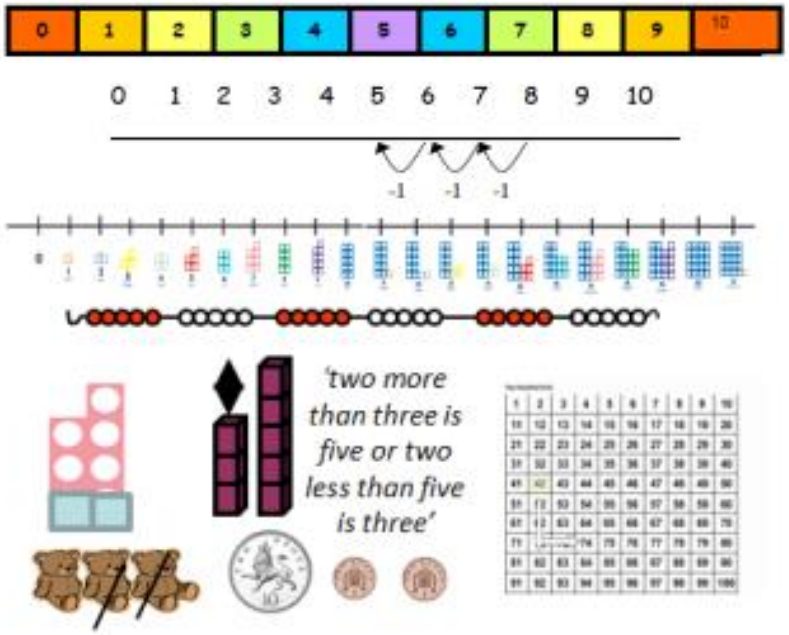
	T	O	.	t
	3	5	.	2
+	1	6	.	0
	5	1	.	2
	1			

SUBTRACTION

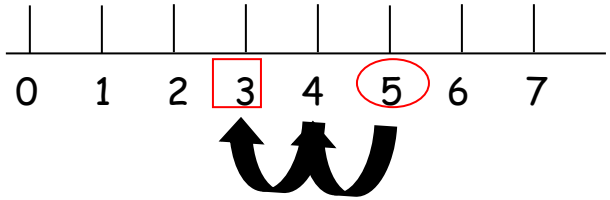
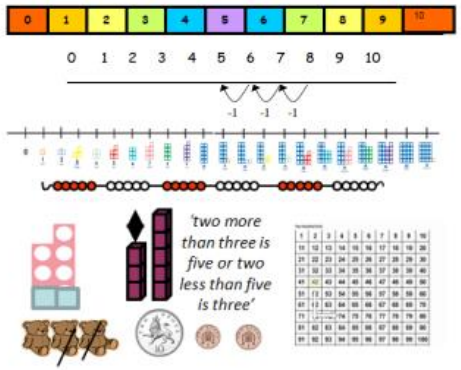
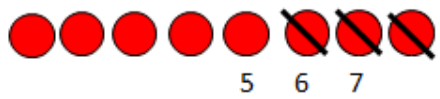
In the Early Years Foundation Stage (EYFS) and Key Stage One children are taught mental calculations which are systematically built upon throughout Key Stage Two.

Children are introduced to the processes of calculation through practical, oral and mental activities, such as counting songs, counting by rote and touch counting as well as through experiencing real-life scenarios, such as buying and selling in a shop. When subtracting, this will include language such as: less (than), fewer (than), take away, total, how many.

Resources that could be used to support this learning:



Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones

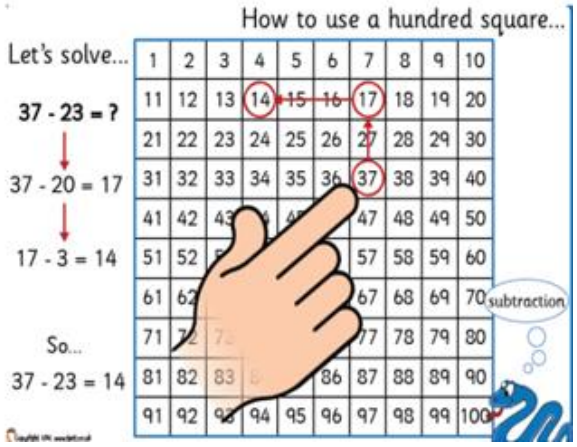
STAGE ONE	
The number line is used and is also supported by the use of the number stick and number beads. *Other resources used to support children with their understanding of number include Numicon and number squares.* Children are taught to subtract larger numbers using the number line as their understanding of number progresses.	<u>Number Line</u> A blank or annotated number line is used for counting back. Children jump on the bottom of the number line when subtracting. For example, 5 – 2 = **<u>3</u>**   Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones Counting back: 8 - 3 = 5  I count 8 counters. I keep 8 in my head. I count backwards 3 as I move 3 counters away. 7, 6, 5

STAGE TWO

When children are ready to be subtracting larger numbers they will do so on a hundred square. They will be encouraged to jump back in 10s and ones.
Children are encouraged to then record this as a number sentence.

Children will continue to use the blank number line throughout each stage as an efficient method of subtraction.

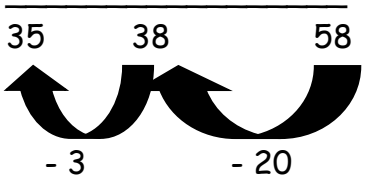
Using a Hundred Square:



Blank number line

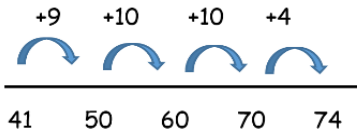
58 – 23 = 35

20 3



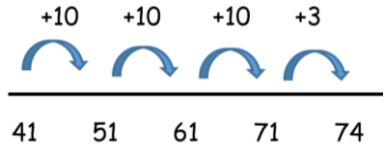
Children will also find the **difference** by counting on from the smaller number (to the nearest ten, then count in tens).

74 – 41



Or, if easier...

74 – 41



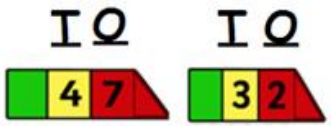
STAGE THREE

Resources used to support children in the expanded method of column subtraction (partitioning in a way that leads to a formal method of subtraction) include place value cards.

Children may use the expanded method of subtraction for calculations involving decimals in the context of money and measures.

Partitioning:

47 - 32 =



Expanded method of column subtraction

Once children are confident with place value and partitioning, they begin to set out subtraction as below to support the formal method.

1 digit = 1 square.

Note the subtraction sign on the LEFT hand side.

47 – 35 = 12

	T			O			
	4	0		7			
-	3	0		2			
	1	0	+	5	=	1	5

With 'exchanging', the ten or hundred etc. is placed as shown. Children must be taught to leave a row and/or a column of squares for this:

72 – 54 = 18

	T			O			
	6	0		12			
	~~7~~	0		~~2~~			
-	5	0		4			
	1	0	+	8	=	1	8

Subtracting *decimals* using column subtraction:

£3.45 - £1.91 = 154p or £1.54

	H		T		O		
	200		140				
	~~300~~		~~40~~		5		
-	100		90		1		
	100		50		4	=	154p

STAGE FOUR

When children are secure with place value, they can begin using the compact method of subtraction (with increasingly larger numbers) with at least one 'exchange'.

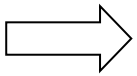
Column subtraction

Always start on the right-hand side with the ones column.

With 'exchanging' the ten or hundred etc., it is placed as shown.

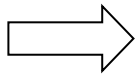
486 – 165 = 321

	H	T	O
	4	8	6
-	1	6	5
	3	2	1



323 – 148 = 175

	H	T	O
	2 ~~3~~	11 ~~2~~	$^{1}3$
-	1	4	8
	1	7	5



Exchange 1 hundred for 10 tens, then 1 ten for ten ones.

3607 - 489

5 9

3 ~~6~~ ~~0~~ ~~7~~

1 1 1

- 489

3118

And then including decimals….

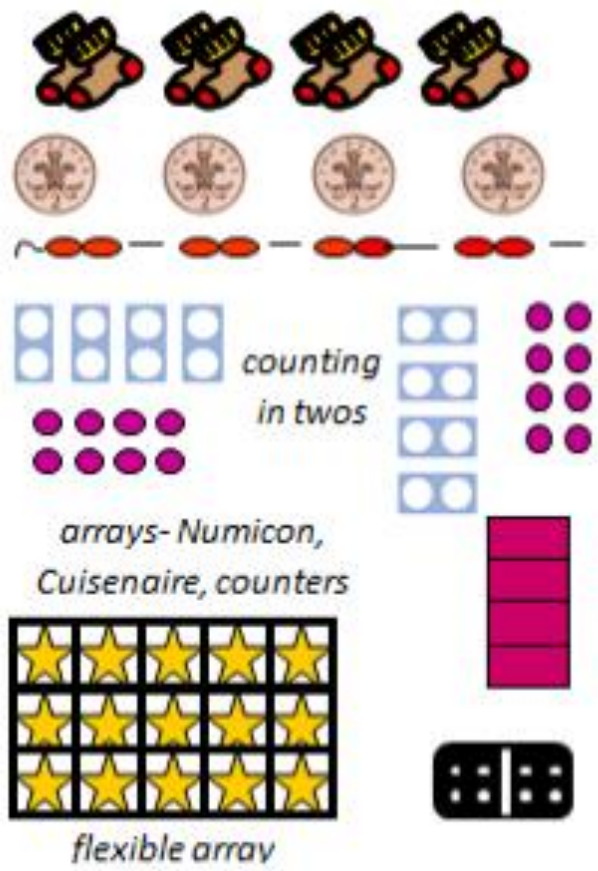
£18.27 – £2.54 = £17.46

	T	O	.	t	h
		1 7			
	~~1~~	~~8~~	.	$^{1}2$	7
-		2	.	5	4
	1	5	.	7	3

MULTIPLICATION

In the Early Years Foundation Stage (EYFS) and Key Stage One children are taught mental calculations which are systematically built upon throughout Key Stage Two.
Children will group objects practically and orally, talking about the total and 'how many lots of' they have. They can multiply with visual objects and pictorial representations. Children need the experience of counting objects in 2s, then 10s and 5s.

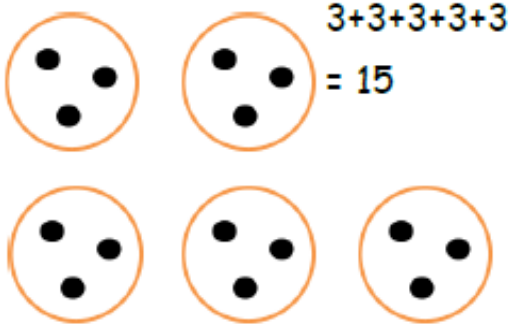
Resources that could be used to support this learning:



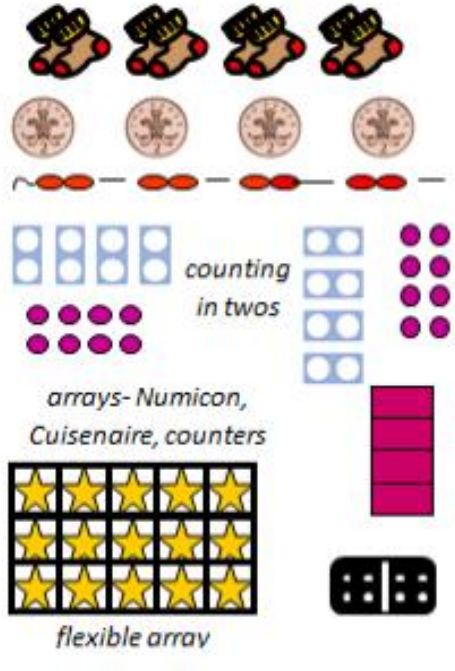
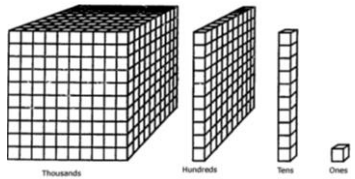
How many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



3+3+3+3+3
= 15

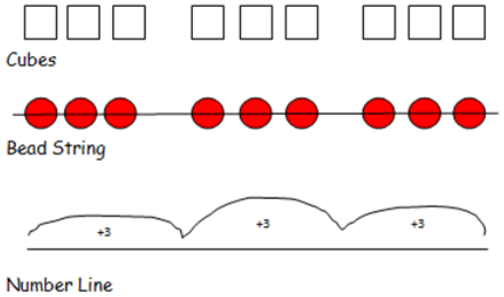
STAGE ONE	
Arrays are used to support repeated addition. *To support this learning, use a range of practical experiences, including outdoor learning.*   *Also, grouping may help the children with the concept of sharing equally between groups.*	<u>Arrays</u> Visual representation of 'groups of' and 'lots of', through practical use of counters, objects and bead strings. e.g. **4 x 2** = 2 + 2 + 2 + 2 = **8** ■ ■ ■ ■ or ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ ■ 4 x 2 2 x 4 There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  3+3+3+3+3 = 15    *Dienes blocks*

STAGE TWO

Blank number lines are used to support repeated addition.

To support this learning, use cubes and bead strings above the number sentence as a visual aid.
e.g.

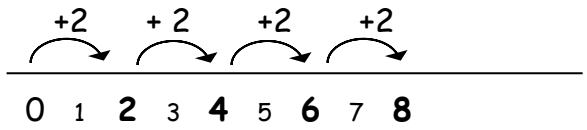
3 x 3 = 3 lots (or groups) of 3 = 9



Arrays should continue to be used as this will support the later learning and development of the grid method.

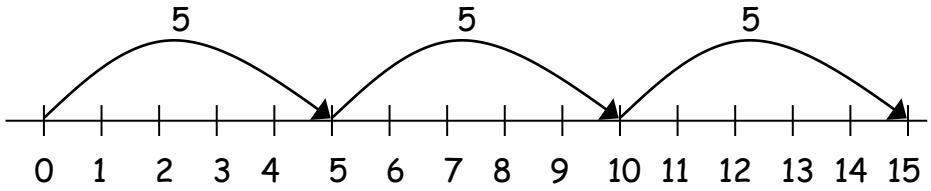
Blank number lines

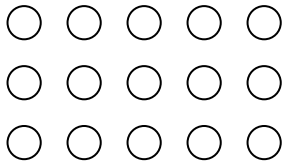
4 x 2 = 4 lots of 2 = 2 + 2 + 2 + 2 = **8**



3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 3 x 5

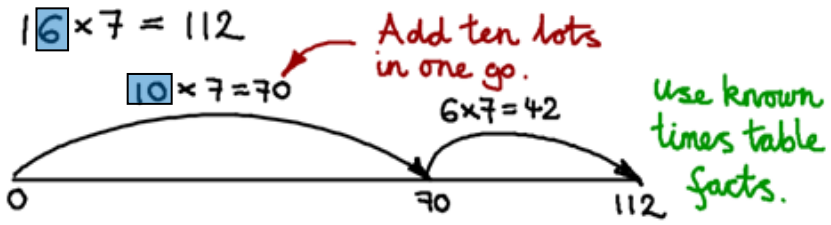
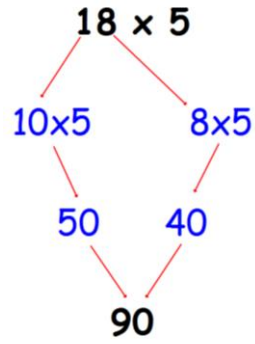
3 x 5 = 5 + 5 + 5 = **15**





3 x 5 = 15

5 x 3 = 15

STAGE THREE		
Resources used to support children in partitioning multiplication include place value cards and Dienes. *To support this learning, partitioning can also be shown on a number line.* When ready, children can record this calculation using brackets or by setting out as shown in the far right diagram.	Partitioning multiplication **14 x 2 = 28** 10 x 2 = 20 4 x 2 = 8  **16 x 7 = 112**  Partitioning multiplication with brackets 18 x 5 = (10 x 5) and (8 x 5) = 50 + 40 = 90	or: 14 x 2 = 10 x 2 = 20 4 x 2 = 8 28 

STAGE FOUR

Once children are confident with place value and partitioning, they begin to use the grid method for multiplication.

To support this learning, frameworks (with the grid method set out) may be provided.

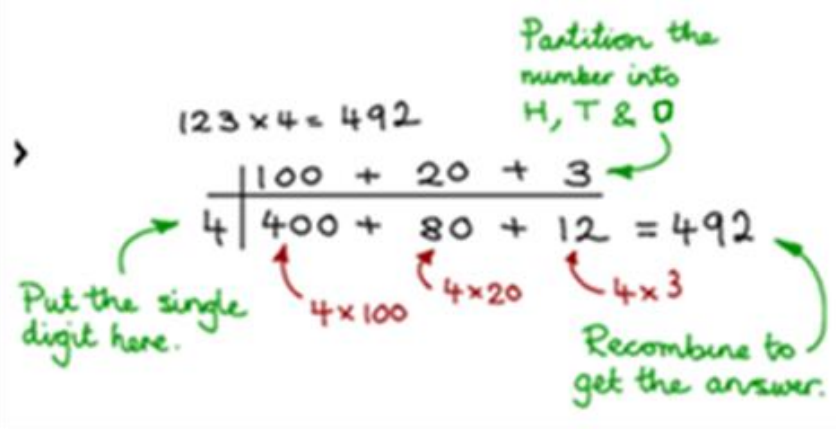
Use resources such as Dienes and place value counters to support the learning of this method in a visual way.

Children will progress using the grid method of calculation using larger numbers and decimals in the contexts of money and measures.

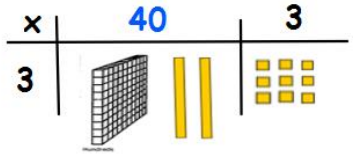
Grid method

43 x 3 = 129		
x	40	3
3	120	9

120 + 9 = 129



43 x 3 = 129



142g x 28 = 3,976g or 3.976kg			
x	100	40	2
20	2000	800	40
8	800	320	16

2,840 + 1,136 = 3,976

24.3sec x 6 = 145.8sec or 2mins 25.8sec			
x	20	4	0.3
6	120	24	1.8

120 + 24 + 1.8 = 145.8

STAGE FIVE

The next step is to progress onto the column method of multiplication.

Extended multiplication method

(Note: the operation signs are always on the left)

138 x 7 = 966

	H	T	O	
	1	3	8	
x			7	
		5	6	(7 x 8)
	2	1	0	(7 x 30)
+	7	0	0	(7 x 100)
	9	6	6	

Lay out calculations using knowledge of place value.

Leave a line below for space when carrying.

STAGE SIX

Short

138 x 7 = 966

	H	T	O	
	1	3	8	
x			7	
	9	6	6	
	2	5		

Carry below the line.

Short with decimals

3.4 x 8 = 27.2

	T	O	.	t
		3	.	4
X				8
	2	7	.	2
	2	3		

Long

56 x 27 = 1,512

	Th	H	T	O	
			5	6	
x			2	7	
		3	⁴9	2	(7x56)
+	1	¹1	2	0	(20x56)
	1	5	1	2	
		1			

Multiply by bottom digits, ones first.

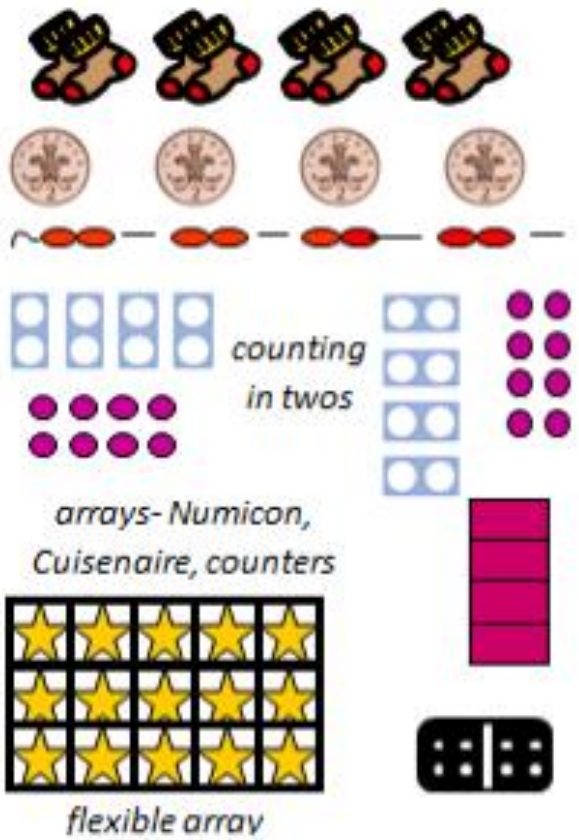
Use '0' as a place holder, representing x by tens.

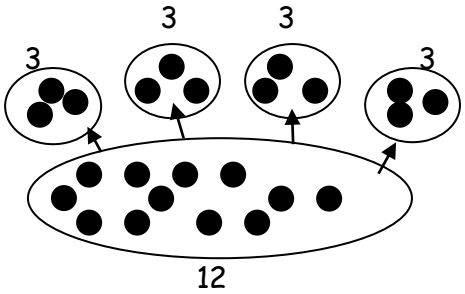
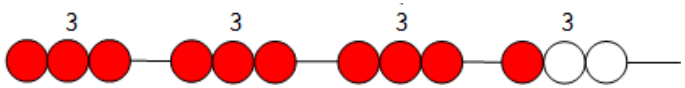
*Carry numbers within the calculation, then **cross them out once added back in.** When adding the two parts of the answer together, carry numbers below the line.*

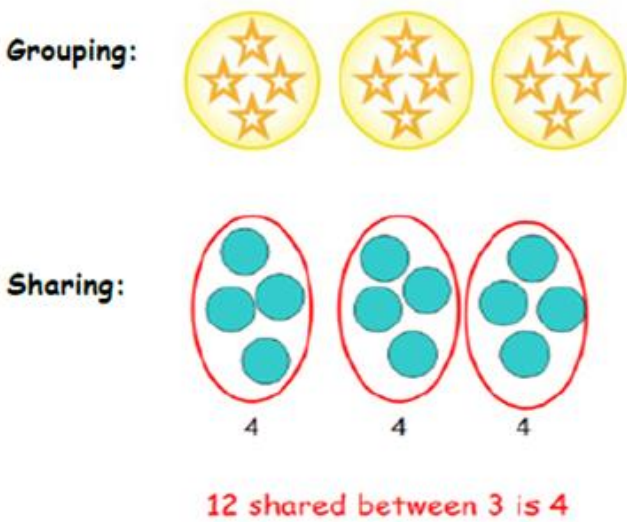
DIVISION

In the Early Years Foundation Stage (EYFS) and Key Stage One children are taught mental calculations which are systematically built upon throughout Key Stage Two. Children will group and share objects practically and orally, talking about the total and 'how many groups of' they have. They can divide with visual objects, Numicon and pictorial representations. Children need the experience of counting objects in 2s, then 10s and 5s. When dividing, this will include language such as: share, fair, divide, equally, left over, groups of.

Resources that could be used to support this learning:



STAGE ONE	
Children use concrete materials for sharing and grouping when they begin work on division. They will begin by sharing in 2s, 10s and then 5s. Children will need to have lots of opportunities to share and group practically and learn the vocabulary of share, fair, divide, equally, left over, groups of.	<u>Sharing</u> 12 shared between 4 = 3  Bead strings can help children interpret division calculations such as 10 ÷ 3 as 'how many 3s make 10?' 



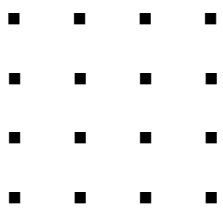
STAGE TWO

Arrays are used to support division as grouping as well as sharing (which is also how multiplication is taught using 'groups of').

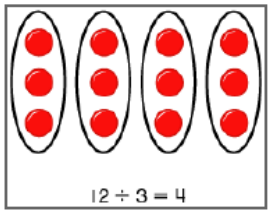
To further children's learning, symbols are used for unknown numbers. This will make clear the relationship between division and multiplication.

Arrays

16 ÷ 4 =



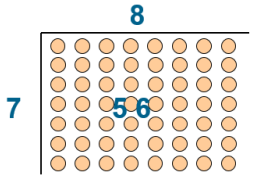
Arrays:



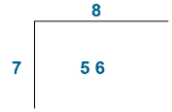
This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

56 ÷ 7 = 8



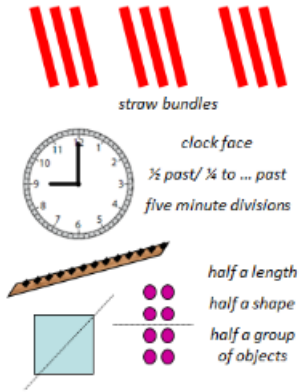
Which will eventually support the transition towards later stages…

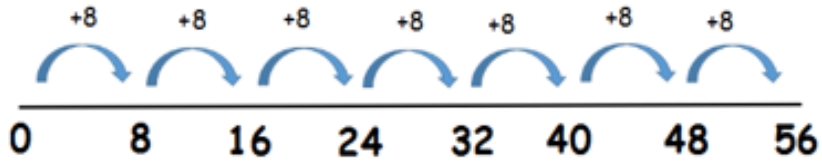


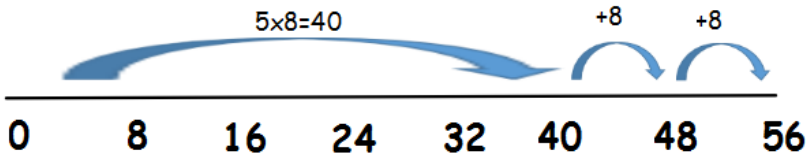
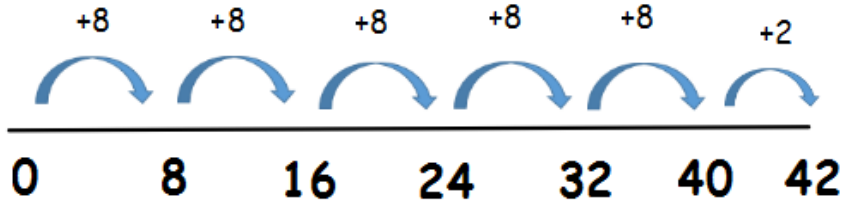
Using symbols to stand for unknown numbers to complete equations using inverse operations.

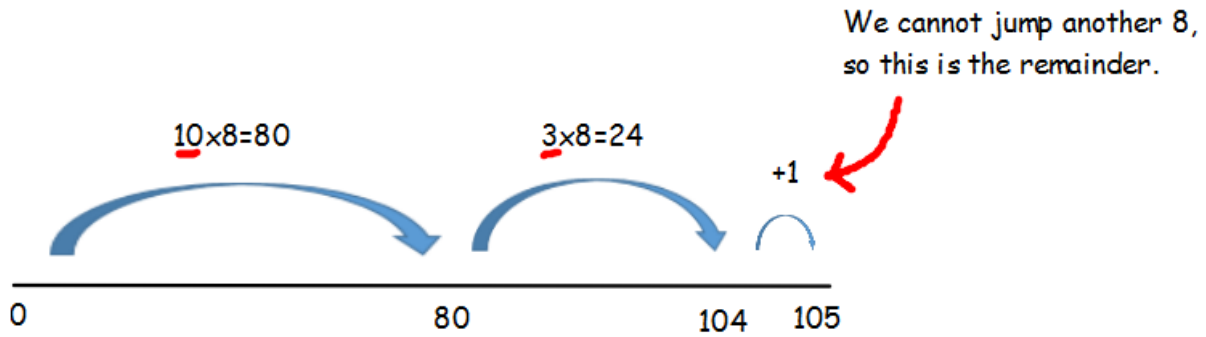
□ ÷ 2 = 4 20 ÷ △ = 4 □ ÷ △ = 4

Resources to support learning:



STAGE THREE	
Understanding division as repeated addition is key to understanding formal methods of division.	**56 ÷ 8 = 7** How many times does eight 'go into' 56? It fits in 7 times exactly. To develop speed and effeciency, encourage use of times table facts: eg 7x8=56 

STAGE FOUR	
By using known times table facts shortcuts can be taken to reduce the number of steps. Children then progress onto dividing with remainders.	**56 ÷ 8 = 7** Jump 5 lots of 8 in one go, then jump in 8s until no longer possible. Or, if able, use exact times table fact, eg 7x8=56.  **42 ÷ 8 = 5 r2** 

STAGE FIVE	
Using known times table facts allows children to jump in larger multiples or 'chunks'. Children will be performing division where there are remainders.	**105 ÷ 8 = 13r1** 

STAGE SIX

Grouping on a blank number line (stage five) leads to vertical format of grouping method.

The number of steps decreases as children's estimates improve. This includes using known number facts to support bigger calculations – eg; if I know 3x6=18, then I know that 30x6=180 (as seen in far right diagram).

Grouping method

56 ÷ 4 = 14

		1	4			
	4	5	6			
	-	4	0	10 groups		
		1	6			
	-	1	6	4 groups		
			0			

Add up the groups they have taken away to reach the answer: 14

Grouping method with remainders

196 ÷ 6 = 32 r4 (or **32 $^{4}/_{6}$** or **32 $^{2}/_{3}$**)

			3	2			
	6	1	9	6			
	-	1	8	0	30 groups		
			1	6			
	-		1	2	2 groups		
				4			

Add up the groups they have taken away
To reach the answer: 32 r 4

The same method applies to larger numbers.
977 ÷ 36 = 27 r5

				2	7	r 5
3	6		9	7	7	
		-	7	2	0	20 groups
			2	5	7	
		-	1	8	0	5 groups
				7	7	
		-		7	2	2 groups
					5	(remainder)

STAGE SEVEN

Long division is an extension of the grouping method.

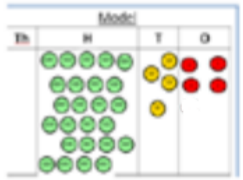
This can be a time consuming stage but is designed to illustrate what is happening to the numbers in terms of place value.

In addition to using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

<u>As soon as pupils have understood, move on to the shorter abstract methods.</u>

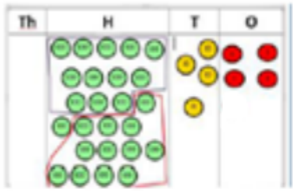
2544 ÷ 12
How many groups of 12 thousands do we have?
None

Exchange 2 thousand for 20 hundreds



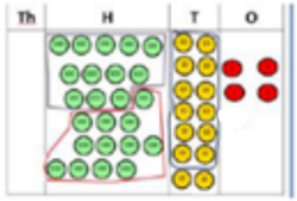
$$12\overline{)2544}\quad 0$$

How many groups of 12 are in 25 hundreds? 2 groups. Circle them.
We have grouped 24 hundreds so can take them off and we are left with one.



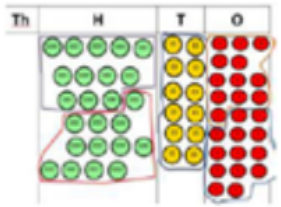
$$12\overline{)2544}\quad 02,\ 24,\ 1$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$12\overline{)2544}\quad 021,\ 24,\ 14,\ 12,\ 2$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$12\overline{)2544}\quad 0212,\ 24,\ 14,\ 12,\ 24,\ 24,\ 0$$

Here is the same style of calculation again without the counters.

The zero at the beginning of the answer is not needed but is included to illustrate that 13 does not go into 3, so the 3 was carried to create 30 (3 thousands = 30 hundreds). The corresponding highlights show how the 'lots of' 13 (and the remainder) create our final answer.

3,017 ÷ 13 = 232 r1

			0	2	3	2	r1	
	1	3	3	0	1	7		
		-	2	6	↓	↓		
				4	1			
			-	3	9	↓		
					2	7		
				-	2	6		
						1		

Take care to add the '0' if the divisor doesn't divide into the dividend.

9,362 ÷ 12 = 780 r2

			0	7	8	0	r2		
	1	2	9	3	6	2			
		-	8	4	↓	↓			
				9	6				
			-	9	6	↓			
					0	2			

Long division

432 ÷ 15 becomes

			2	8	r 12
1	5	4	3	2	
		3	0	0	
		1	3	2	
		1	2	0	
			1	2	

Answer: 28 remainder 12

432 ÷ 15 becomes

			2	8	
1	5	4	3	2	
		3	0	0	15×20
		1	3	2	
		1	2	0	15×8
			1	2	

$\frac{12}{15} = \frac{4}{5}$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

			2	8 ·	8
1	5	4	3	2 ·	0
		3	0	↓	↓
		1	3	2	
		1	2	0	
			1	2	0
			1	2	0
					0

Answer: 28·8

These examples are taken from the national curriculum and demonstrate remainders as fractions and decimals.

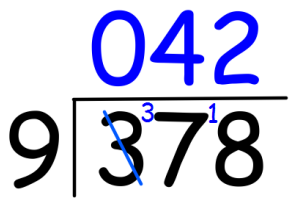
STAGE EIGHT

Short division is an extension for children who have a secure knowledge. It is not an alternative to the grouping or long division method.

Compare this with the longer version in stage 7 to see the individual calculations behind this shortened version.

Short division

378 ÷ 8 = 42



3,017 ÷ 13 = 232 r1 = $232^{1}/_{13}$

				2	3	2	r1	
	1	3	3	0	$^{4}1$	$^{2}7$		

Start on the left. Carry remainder in front of next digit. Answer above the line.